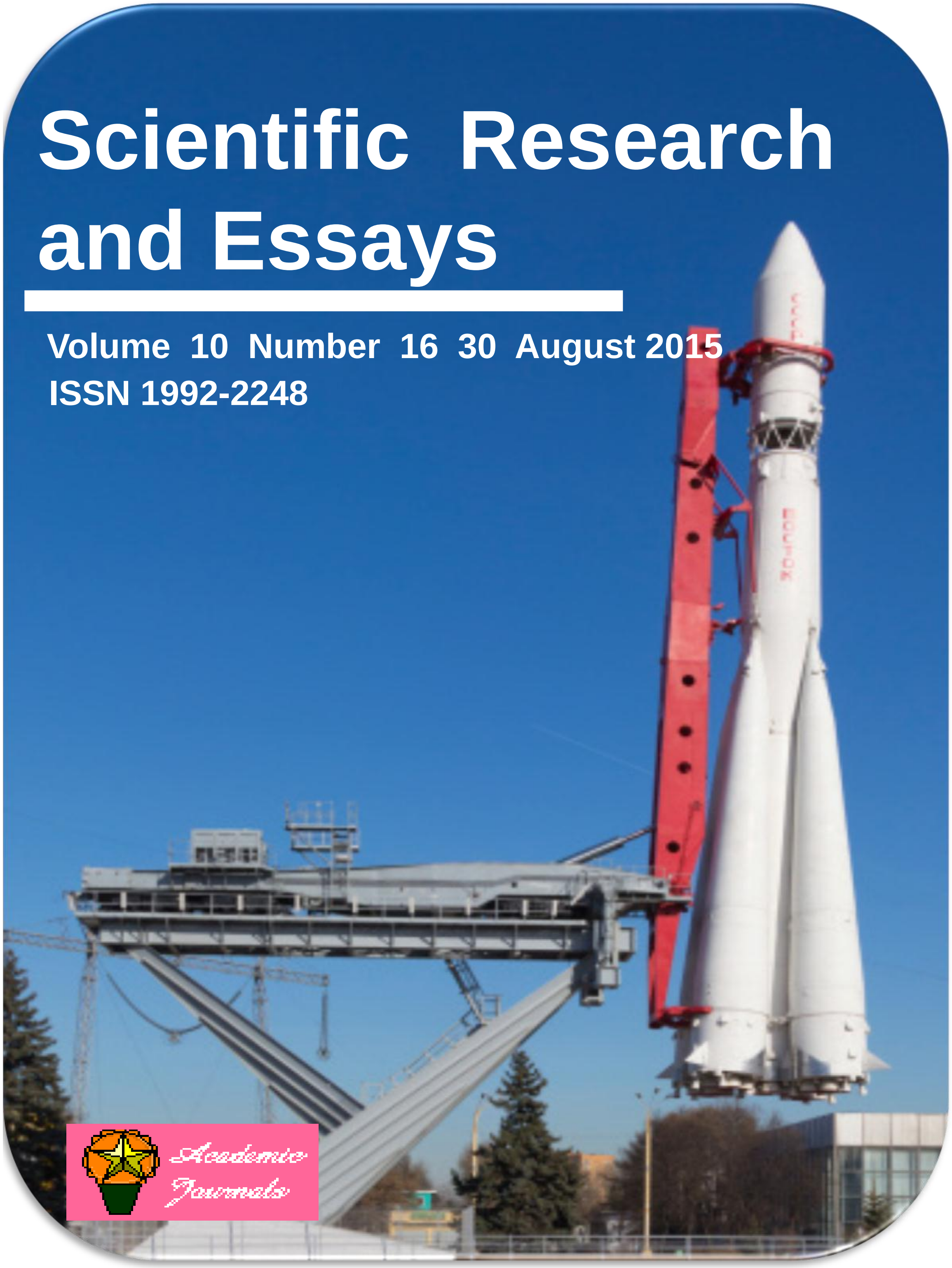
Scientific Research and Essays

Volume 10 Number 16 30 August 2015
ISSN 1992-2248



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academicJournals

Vol. 10(16), pp. 513-521, 30 August, 2015
DOI: 10.5897/SRE2015.6217
Article Number: 5D80E2354960
ISSN 1992-2248

http://www.academicjournals.org/SRE

Scientific Research and Essays

Full Length Research Paper

Air pollution indicators in Brazil, Russia, India and China (BRIC) countries

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Received 27 March, 2015; Accepted 30 July, 2015

Over the past several decades, the amount of attention given to various countries' environmental impact has greatly increased. Brazil, Russia, India and China (BRIC) have been drawing special attention due to the pollution emissions released into the atmosphere by their increasing number of industries and their exaggerated consumption of products. This article aims to elucidate and analyze the evolution of some of the atmospheric indicators of the BRIC group of countries, the amount of money each country invests in research and development of renewable energies, and the possible human health consequences of excess exposure to CO_2. Secondary data on atmospheric indicators of the BRIC group of countries were obtained and critically analyzed. They were first tabulated in an Excel spreadsheet and then presented in tables and figures. Linear regression and the correlation between CO_2 and global warming for the next few years were also determined. The findings reveal that CO_2 emissions per capita as well as the kilograms in USD$ of the GDP of the countries showed an average increase of 15% in Brazil, Russia, and India. The average increase in China was 30%. China and Brazil are the countries that invest the most in research and development. It is concluded based on the forecasted predictions that if the surveyed countries adopt effective preventive measures, the CO_2 emissions and amount of air pollution could show a downward trend over time; on the other hand, if nothing is done to reverse this situation, the indexes may even exceed the forecasts.

Key words: Brazil, Russia, India and China (BRIC), carbon dioxide, air pollution.

INTRODUCTION

The acronym BRIC (Brazil, Russia, India and China) was coined by the English economist, Jim O´Neill in 2001. At the time of his study, he noted that these four developing countries showed an economic growth rate that was higher than the average rate of other developed countries, such as the United States, Japan and Germany (O´Neill,

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2001). From that point on, this classification began to be disseminated and used by other authors and scientists (Bharadwaj, 2006; Pao and Tsai, 2011; Bloom, 2000).

Since the creation of the acronym, the BRIC countries have been forming an informal alliance as a result of the growth of their political and economic power. This means that even if these countries are not organized as an economic bloc, they create economic cooperation, which provides incentives for foreign, direct investment and reciprocal fortification of their economies. In fact, the BRIC countries have the four largest economies outside the Organization for Economic Cooperation and Development (OECD) (Cheng et al., 2007; Ferraz, 2013).

It is known that economic growth happens through industrial growth; thus, it requires the use of natural resources. The consequences of BRIC´s rapid economic growth and development have been drawing the attention of several environmentalists that are concerned about the high level of pollution emissions these countries release into the atmosphere (Abramovay, 2010) as a result of the increased number of industries and the exacerbated consumption of more products that are unsustainable (Gonçalves-Dias and Moura, 2007; Ekins, 1991; McGregor, 2005), which take a long time to decompose; require an extensive recycling process, or even compromise natural resources (Tukker et al., 2010).

The consequences of this lack of environmental awareness have been so alarming that several scientific journals featured this subject on their front covers in 2014. The articles in most of those journals presented alarming information and data related to climate change. All in all, if radical revision of sustainable actions is not undertaken, the environmental prediction for future generations is not optimistic.

There are several ways to define and understand sustainability. In scientific academia, sustainability consists of two main elements: the triple bottom line and time, namely environmental, economic, and social dimensions over a specified period. In the social understanding, sustainability is generally comprehended as the resources created by humans for environmental protection; in other words, the concept only has an environmental analogy. Nevertheless, sustainability ends up referring to other contexts, such as public administration, regional government policies, and business management (Mawhinney, 2002; Ruscheinsky, 2004; Böhringer and Jochem, 2004; Dias, 2011).

In the 21st century, the problems of sustainability play a central role in the push toward industrial and economic growth at any cost (Jacobi, 2003). In this context, the BRIC countries are not only responsible for how they treat the modernization of their industries and economies, as previously mentioned, but also responsible for the sustainability of this process as a whole, given that the raw materials to make many products are taken from the natural world.

Dias (2006) warned that, in emerging countries, the amount of waste only grows with the increase in the consumption of both goods and products. It is up to the citizens and governmental authorities to think of conscious alternatives regarding the reuse of materials, recycling, or even the correct disposal of waste. After all, the lack of proper waste management can be harmful to human health, in some cases. In other cases, indirectly related to the lack of waste management, global warming of the Earth may increase due to the elevated fossil fuels used in industrial production. This causes severe damage to the ozone layer, which can severely impact human health, increasing the incidence of skin cancer (Netto, 2009).

The reports published in recent years by the United Nations Environment Programme (UNEP) note that the BRIC countries and the United States are the main CO_2 emitting countries. Others consider China to be the largest emitter of CO_2, which is primarily responsible for the rising temperatures associated with climate change. However, the Intergovernmental Panel on Climate Change (IPCC) report for 2014 noted that China is also the country that has invested the most in researching and developing renewable sources; in 2013 alone, it invested USD 56 billion into renewable energy research.

According to the 2014 Environmental Performance Indicators (EPI), the top 10 carbon dioxide (CO_2) emitting countries contribute 78% of the total carbon dioxide that is emitted globally (Yale Center for Environmental Law and Policy, 2015).

The level of air pollution, especially in large urban centers, raises serious concerns about its impact on the health of people. According to the World Health Organization (2011), children, the elderly, and people with cardiorespiratory problems are the hardest hit. The suspended dust particles of toxic gases found in air pollution contribute to respiratory infections, pulmonary obstruction, and the occurrence of other diseases, such as cancer. Therefore, this work aims to identify and discuss recent atmospheric pollution indicators, stressing CO_2; it also seeks to address the amount of investments that these four countries have made to fund the research and development of renewable energies and other relevant information about this topic.

METHODOLOGY

The study presented in this article collected atmospheric indicators from BRIC countries between 2000 and 2010. The following topics were chosen: air quality and health impact, both of which are Yale EPI profiles.

These indicators were collected from The World Bank (2015), tabulated in an Excel spreadsheet, and then analyzed. After the secondary data were analyzed, they were compiled into tables and figures for better understanding. This study also used the statistical technique of simple linear regression to analyze the relationship between the dependent and independent variables. Normally, there are correlations between these variables and their degree of importance is related to the impact that each of the variables has on the process being examined. According to Dominguete et al. (2006), a correlation is the relationship between two variables. The data can be represented by ordered pairs (x, y), where x is the independent variable (or explanatory) and y is the dependent variable.

The generic model is given by Equation (1), according to Montgomery and Runger (2012), when applied to a sample of size n:

$$\hat{y} = \beta0 + \beta1.X1 + \mathcal{E} \quad (1)$$

Where: $\hat{y}$ = the dependent variable or explained; β_0 = Interceptor or independent variable term; β_1 = *Y* tilt in relation to the variable X_1; X_1 is the independent variable; and Ɛ = random error in *Y*.

The correlation coefficient varies between the limits, – 1 and 1; therefore, it can be positive or negative ($-1 \le r \le 1$). When the correlation coefficient is zero, it means that there is no relationship between the variables. When the correlation coefficient is equal to -1 or +1, a perfect relationship exists between the variables.

The degree of relationship between the variables, which expresses how the variables are related to each other, is defined numerically by the correlation coefficient, represented by (r):

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{(X_i - \bar{X})^2 (Y_i - \bar{Y})^2}} = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y} \quad (2)$$

Where: $\bar{x}$ = is the average of the independent variable *X*; $\bar{y}$ = is the average of the dependent variable *Y*; $\hat{\sigma}xy$ = is the sample covariance between *X* and *Y*; $\hat{\sigma}x$ = is the sample standard deviation of *X*; and $\hat{\sigma}y$ = is the sample standard deviation of *Y*.

Correlation coefficient parameters are as follows:

r = 0 = No relationship
0 < r ≤ 0.30 = Weak relationship
0.30 < r ≤ 0.70 = Average relationship
0. 70 < r ≤. 0.90 = Strong relationship
0.90 < r ≤ 0.99 = Very strong relationship
r = 1 = Perfect relationship

The tabulated data were subjected to simple linear regression, which individually analyzes the data over time and projects future values from the mathematical model (equation), verifying the degree of correlation (Equation 2). In this research, historical data of CO_2 emissions were considered.

As shown in Table 1, the statistics demonstrate that the tabulated data feature a good degree of correlation and that the mathematical model can be used to estimate the future values for both CO_2 emissions and air pollution.

RESULTS AND DISCUSSION

The increase in the level of CO_2 emissions over the 10 year-period (2000 to 2010) varies substantially among the analyzed countries. This is associated with the economic growth of each country (Santana, 2012). CO_2 emissions and the greenhouse effect are directly related, so the data presented in Tables 2 and 3 were analyzed together. The data obtained in the survey are illustrated in Tables 2 and 3.

The increase in CO_2 emissions over that 10-year period varies substantially among the countries analyzed. Brazil, Russia, and India had an average increase of 15%, while the increase in China was 30%. This difference is due to the fact that China has the highest economic growth of the four BRIC countries. According to Naughton (2007) and Huang (2008), the boom in the number of multinational industries establishing enterprises in China occurred in the 1990s, increasing the country's source of revenue. The resulting increase in higher buying power enabled the people of China to have more access to means of transportation and to acquire more durable goods, such as stoves and automobiles, which are responsible for CO_2 emissions.

As seen in Table 2, the country with the highest emission growth per capita is Russia, followed by China, Brazil, and India. In 2010, the CO_2 emissions per capita in Russia were 12.23, while in China it was 6.19. However, it is necessary to emphasize that the CO_2 emissions rate is given per capita, that is, it is based on the total population. As China is the most populous country of the four studied countries it shows a lower CO_2 emissions rate than Russia. According to the World Population Data Sheet (2015), in 2010 China had a population of 1,330.141.295 (1st most populous country in the world), while Russia had a population of 139,390,205 (9th most populous country in the world).

Some scholars (Rohde, 1990; Lashof and Ahuja, 1990; Jenkinson et al., 1991; Dietz, 1997; Fearnside, 1997; Shine et al., 2005) consider the elevation of CO_2 levels in the atmosphere to be largely responsible for the intensification of the greenhouse effect. In historical terms, this elevation is attributed to the burning of fossil fuels (coal, oil, and natural gas) for power generation and, secondarily, to agricultural processes and the destruction of natural vegetation, such as forests (Andreae, 1991; Bodansky, 2001).

Of all four of the BRIC countries, China has the greatest amount of greenhouse gas emissions. In 2000, China emitted the equivalent of 3.41 million particles and that total gradually increased until 8.27 million particles were emitted in 2010; this is an increase of 4.86 million particles within a 10-year period. Over the course of decade, that country more than doubled its equivalent CO_2 greenhouse effect. Brazil´s greenhouse gas emissions grew to 0.09 million particles in 10 years. Russia showed an increase of 0.18 million particles and India showed an increase of 0.82 million particles.

The numbers presented above engender questions such as: How can Russia be the country that emits the most CO_2, but causes a lower greenhouse effect than China? The answer to this is found in the differences in

Table 1. Linear regression data.

	Brazil		China	
	CO_2 emissions	Air pollution	CO_2 emissions	Air pollution
R^2	0.877	0.741	0.991	0.949
Correlation	0.769	0.547	0.981	0.901

Source: Authors (2015).

Table 2. CO_2 Emissions from 2000 to 2010.

Years	Country							
	Brazil		Russia		India		China	
	Per Capta	Kg per US$ of GDP	Per Capta	Kg per US$ of GDP	Per Capta	Kg per US$ of GDP	Per Capta	Kg per US$ of GDP
2000	1.88	0.43	10.63	2.75	1.14	1.97	2.70	2.40
2001	1.91	0.43	10.67	2.61	1.14	1.91	2.74	2.27
2002	1.85	0.42	10.71	2.49	1.14	1.87	2.89	2.21
2003	1.77	0.40	11.09	2.40	1.17	1.81	3.51	2.46
2004	1.84	0.40	11.15	2.23	1.21	1.77	4.08	2.61
2005	1.87	0.39	11.29	2.11	1.25	1.69	4.44	2.57
2006	1.85	0.38	11.72	2.02	1.32	1.65	4.89	2.52
2007	1.91	0.37	11.73	1.86	1.39	1.61	5.15	2.34
2008	2.02	0.38	12.09	1.82	1.54	1.74	5.31	2.21
2009	1.90	0.36	11.09	1.81	1.67	1.76	5.78	2.21
2010	2.15	0.38	12.23	1.91	1.67	1.62	6.19	2.16

Source: The World Bank (2015).

Table 3. Greenhouse effect equivalent to CO_2 in millions (from 2000 to 2010).

Years	Country			
	Brazil	Russia	India	China
2000	0.33	1.56	1.19	3.41
2001	0.34	1.56	1.20	3.49
2002	0.33	1.56	1.23	3.69
2003	0.32	1.61	1.28	4.53
2004	0.34	1.62	1.35	5.23
2005	0.35	1.67	1.41	5.79
2006	0.35	1.67	1.50	6.41
2007	0.36	1.71	1.61	6.79
2008	0.39	1.72	1.81	7.05
2009	0.37	1.57	1.98	7.69
2010	0.42	1.74	2.01	8.27

Source: The World Bank (2015).

the geographical areas of those two countries, taking into account that the greenhouse effect is determined by the radiation over a specific region or territory. According to the World Bank (2015), Russia has a geographical area of 17,075,400 km² (the greatest territorial region in the world), while China has an area of 9,596,960 km^2, that is

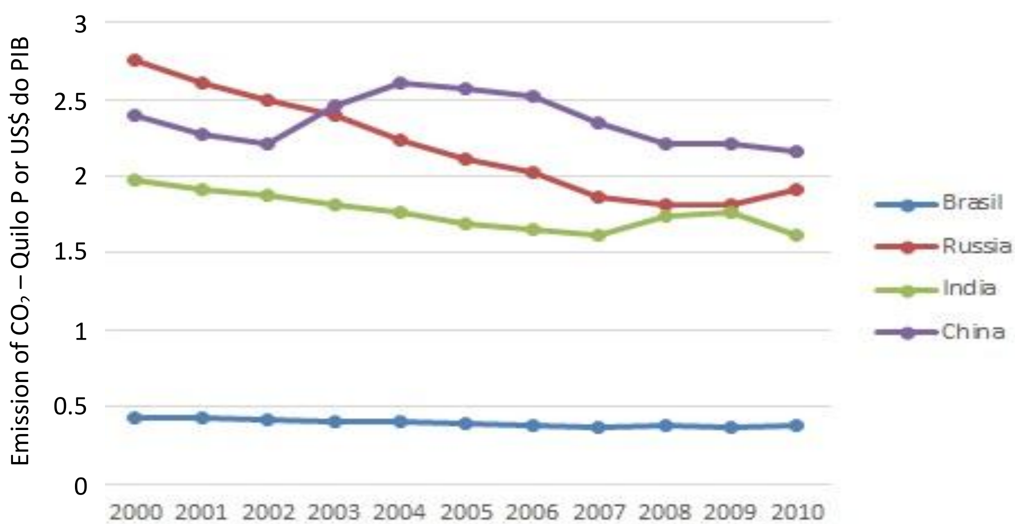

Figure 1. CO_2 Emissions: Kg per USD$ of the GDP (2000 to 2010). Source: Authors (2015).

almost half the size of Russia.

Figure 1 provides a more detailed overview of the kilogram per USD$ in relation to the gross domestic product (GDP). Russia had the largest drop, while the decreases in China and India were constant.

However, as this index depends on the GDP, this suggests that the growth in the amount of pollution is slower than the growth of the GDP. Thus, countries that are growing economically continue to pollute, but the increase in the amount of pollution is lower than the GDP growth.

Subsequently, the CO_2 emission analysis in this study aims to check the percentage of renewable energy use in each country. The results are shown in Table 4.

Among the four countries, Russia was found to have used the least amount of renewable energy. From 2000 to 2010, the country's percentage of renewable energy use was only 1.04%. Thus, its non-renewable energy use was approximately 99%. Henry and Sundstrom (2007) credited this lack of interest in sustainable projects to two main causes: The government's oil extraction projects and the lack of a legal and institutional framework directed toward the causes and effects of environmental pollution.

In 2000, China's renewable energy usage was 17.53%; in 2010 it dropped to 8.49%. However, this scenario is changing because of the critical levels of air pollution in that country and the political pressure from other countries, especially European countries, to export Chinese products at the expense of dirty energy (Vichi, 2009).

Brazil was the only country that used more renewable energy over the course of the 10-year period, which shows that the country is looking for alternatives for cleaner energy production, sometimes triggered by economic benefits or lower operating costs (Vichi, 2009). Goldemberg et al. (2005) justified Brazil's greatest potential for renewable energy usage as being the result of its geography and natural resources. Wind power is generated by the geographic formation of winds in the country's southern region.

This study also investigated the percentage of GDP each of the four countries was willing to spend on research and development to reduce the levels of polluting gases as showed in Table 5.

China and Brazil had the highest percentage of GDP invested in the research and development of new alternative energies. Russia and India had the lowest percentage of investment: 0.08 and 0.06%, respectively.

Research on projects that mitigate the amount of CO_2 emitted into the atmosphere is beneficial to the environment and people's health. The World Health Organization (2011) claimed that, in major urban centers, CO_2 emissions have a serious impact on human health. Zielinski et al. (1997) noted that children, the elderly, and people with respiratory diseases, such as asthma and respiratory insufficiency, are the most affected. Accordingly, this present study sought to demonstrate the evolution of the percentage of children under the age of 5 who suffer from respiratory infections as showed in Table 6.

The World Bank found that 70% of children in India under the age of 5 had acute respiratory infection in 2010 that resulted from air pollution, whereas in Brazil it was 50%. Because India has a high greenhouse effect and a high non-renewable energy use (75.12%), it was already expected that the incidence of respiratory infections in children would be higher in that country than in Brazil

Table 4. Percentage of renewable and non-renewable energy use (2000 to 2010).

Years	Country							
	Brazil		Russia		India		China	
	Renewable	Non-renewable	Renewable	Non-renewable	Renewable	Non-renewable	Renewable	Non-renewable
2000	24.87	75.13	1.11	98.89	32.56	67.44	17.53	82.47
2001	25.24	74.76	1.09	98.91	32.48	67.52	17.14	82.86
2002	26.75	73.25	1.11	98.89	31.95	68.05	16.20	83.80
2003	28.71	71.29	0.95	99.05	31.55	68.45	14.21	85.79
2004	29.10	70.90	1.09	98.91	30.20	69.80	12.36	87.64
2005	29.40	70.60	1.06	98.94	29.50	70.50	11.47	88.53
2006	29.79	70.21	1.12	98.88	28.47	71.53	10.55	89.45
2007	30.65	69.35	0.99	99.01	28.00	72.00	10.09	89.91
2008	31.56	68.44	0.91	99.09	27.17	72.83	9.75	90.25
2009	31.65	68.35	0.98	99.02	25.22	74.78	9.18	90.82
2010	30.70	69.30	0.99	99.01	24.88	75.12	8.49	91.51

Source: The World Bank (2015).

Table 5. Percentage of GDP spent on research and development (2000 to 2010).

Years	Country			
	Brazil	Russia	India	China
2000	1.02	1.05	0.74	0.90
2001	1.04	1.18	0.72	0.95
2002	0.98	1.25	0.71	1.07
2003	0.96	1.29	0.71	1.13
2004	0.90	1.15	0.74	1.23
2005	0.97	1.07	0.81	1.32
2006	1.01	1.08	0.80	1.39
2007	1.10	1.12	0.79	1.40
2008	1.11	1.04	0.84	1.47
2009	1.17	1.25	0.82	1.70
2010	1.16	1.13	0.80	1.76

Source: The World Bank (2015).

Airborne particles intensify respiratory infections and other diseases, such as cancer. They can penetrate the lungs and get into the bloodstream, thus increasing cardio-vascular problems. This is why many countries monitor the levels of particulate matter (PM), especially PM10 (particles ranging between 2.5 microns and 10 microns in diameter) (Yale Center for Environmental Law and Policy, 2014).

In addition to analyzing CO_2 emissions as well as verifying the greenhouse effect and the consequences caused by air pollution, it is necessary to simulate the long-term quantity of the CO_2 these countries will emit if the present system remains constant. Thus, the authors chose to predict a scenario for China and Brazil, basically because of the availability of data for those two countries. The results are presented in Figures 2 and 3.

Keeping the current scenario of investments in renewable forms of energy and the amount of CO_2 emissions for China and Brazil, the statistics show that atmospheric damage will increase; air pollution will increase because the variables are highly correlated. Although carbon dioxide is not solely responsible for air pollution, this study separated the carbon dioxide variable from the other gases that cause air pollution.

All the results (forecast) for the CO_2 emissions and air pollution levels underwent a simple linear regression

Table 6. Percentage of children under the age of 5 with respiratory infections.

Years	Countries			
	Brasil	Russia	India	China
2000	45.0	Information not avaliable	66.0	Information not avaliable
2001	45.0	Information not avaliable	66.5	Information not avaliable
2002	45.7	Information not avaliable	67.0	Information not avaliable
2003	46.0	Information not avaliable	67.0	Information not avaliable
2004	47.0	Information not avaliable	67.8	Information not avaliable
2005	47.9	Information not avaliable	68.0	Information not avaliable
2006	48.4	Information not avaliable	68.7	Information not avaliable
2007	48.0	Information not avaliable	67.0	Information not avaliable
2008	49.0	Information not avaliable	69.0	Information not avaliable
2009	49.6	Information not avaliable	70.0	Information not avaliable
2010	50.0	Information not avaliable	70.0	Information not avaliable

Source: The World Bank (2015).

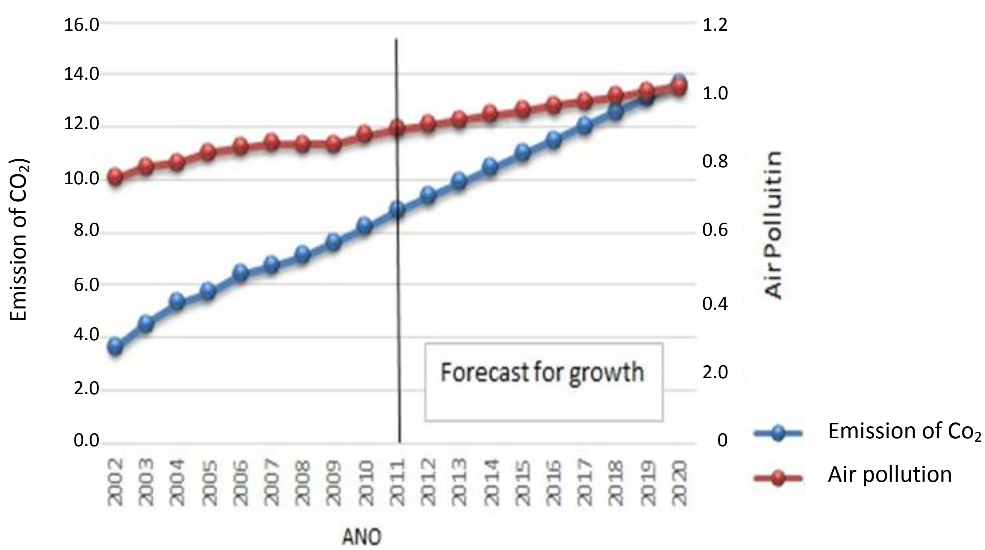

Figure 2. CO_2 emissions forecast for China through 2020. Source: Authors (2015).

technique and the findings support the predictions; the regression and correlation coefficients indicate that the variables are correlated and that the developed mathematical model has a strong positive influence, as previously shown in Table 1.

It is noteworthy that such projections have already indicated a growth trend in recent years for both CO_2 emissions and air pollution. However, if the surveyed countries adopt preventive measures, the CO_2 emissions and the amount of air pollution could show a downward trend, as expected in the forecasted predictions; on the other hand, if nothing is done to reverse this situation, the indexes may even exceed the forecasts.

Conclusion

It is undeniable that the environment, energy use, and economic growth are intertwined. Together they contribute to sustainability, which can be measured by a country's air quality, its use of natural resources, and its quality of life. Therefore, Brazil, Russia, India, and China are rethinking their current models of energy; and, although little is reversed by the public sector's investment in renewable energy, as shown in this study, the environment (through climate change and other indicators) has shown signs that the current model of production and consumption is not sustainable in the

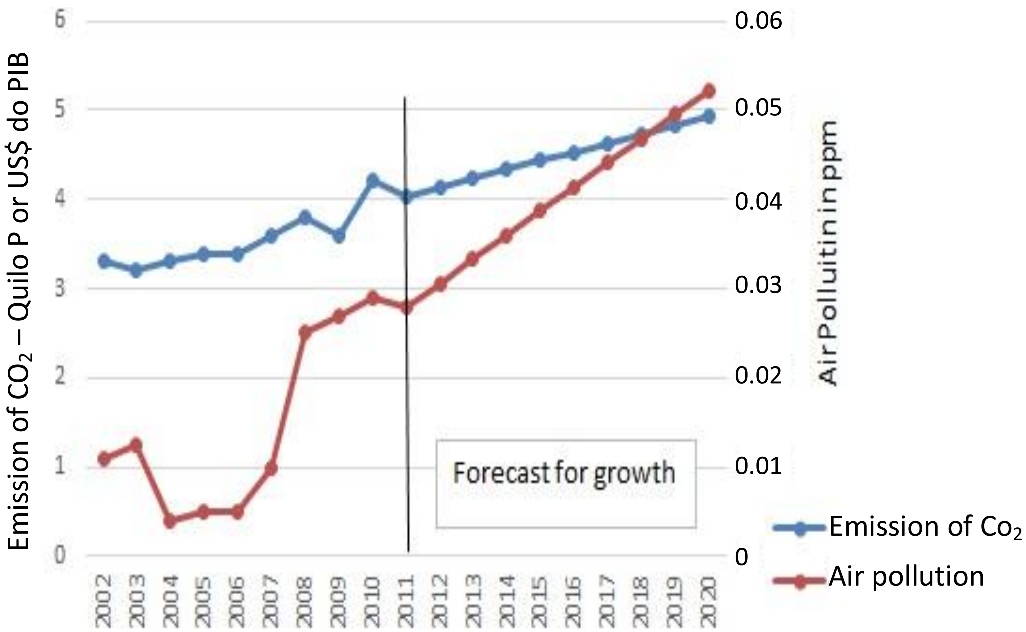

Figure 3. CO_2 emissions forecast for Brazil through 2020. Source: Authors (2015).

long-term. This study also pointed out that polluting gases, especially carbon dioxide, are significantly increasing the incidence of respiratory diseases in young children. The IPCC (2014) has also noted that, since the 1950s, many studies have observed the environmental changes that have taken place on the planet: the atmosphere and the oceans have warmed, the ice glaciers have melted, and the sea levels have risen. Taken together, these factors underscore the interconnectedness of Earth's ecosystem. Indeed, the BRIC countries still face a great challenge as they continue to elaborate on their sustainable development strategies.

Conflict of Interest

The authors have not declared any conflict of interest.

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academicJournals

Vol. 10(16), pp. 522-530, 30 August, 2015
DOI: 10.5897/SRE2015.6305
Article Number: A1A16BC54962
ISSN 1992-2248

http://www.academicjournals.org/SRE

Scientific Research and Essays

Full Length Research Paper

Interval study of convergence in the solution of 1D Burgers by least squares finite element method (LSFEM) + Newton linearization

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Received 3 August, 2015; Accepted 14 August, 2015

This paper aims to apply the least squares finite element method (LSFEM) in conjunction with the linearization technique known as Newton method. For this, the application chosen was the traditional 1D Burgers equation. From two numerical applications and an error analysis based on knowledge of the exact solution, will be possible to present convergence ranges for this proposal.

Key words: Least squares finite element method (LSFEM), Burgers equation, Newton method, convergence range.

INTRODUCTION

The vast majority of physical problems is governed, or may be represented by Partial Differential Equations. Some mathematical methods are capable of producing analytical solutions of physical problems, more precisely of heat and mass transfer problems (Arpaci, 1966; Bejan, 1996; Carslaw and Jaeger, 1986), but only of some, and problems very simplified. So it is taken as a fundamental tool for solving heat and mass transfer problems the numerical methods. For decades, numerical methods have been used to solve such problems, among them stand out the finite difference method (Smith, 1971), finite volume (Chung, 2002) and finite element (Lewis et al., 2004; Donea and Huerta, 2003; Reddy, 1993).

Since the start of the 50's with Turner et al. (1956), Clough (1960), Argyris (1963), Zienkiewicz and Cheung (1965), Oden and Wellford (1972), the finite element method has been used with great success in various branches of engineering.

In Donea and Quartapelle (1992) the authors present a finite element method to solve transient problems governed by linear or nonlinear equations with dominant advective terms. The first is the Generalized Galerkin Method, which provides excellent results due to a correct relationship between the spatial and temporal variations expressed by the theory of characteristics. Beyond that show that the least square method used shows the simplicity of the Taylor-Galerkin method and the unconditional stability of methods of characteristics, however, its accuracy is committed to numbers Courant larger than unity.

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It is important to highlight the work of Burrel et al. (1995) where the authors present the numerical solution via LSFEM of a pollutant transport case in a one-dimensional domain and the work of Bramble et al. (1998) which introduce and analyze two least squares method for elliptic differential equations of second-order with mixed boundary conditions.

Vujicic and Brown (2004) show numerical solution of a three-dimensional transient heat conduction case, in which several time discretization methods have been tested, among which especially the method value β where β is admitted to 0, 0.5, 0.75 to 1 besides using of LSFEM. Whereas Romão and Moura (2012) discuss how numerically solving the partial differential equation modeling convection-diffusion-reaction/heat generation in 3D domain with variable coefficients using the Galerkin method and the LSFEM.

In a new paper, Romão et al. (2011) again compare the results obtained from the two of the FEM variants: LSFEM and Galerkin method when applied to the Poisson equation and the Helmholtz equation. The analysis is made in Poisson's equation applied to diffusion in solids and in the Helmholtz equation applied to diffusion in solids with generation. As well as at work Romão and Moura (2012), the L_2 and L_∞ errors were also calculated and the values for each variant of the FEM, compared between themselves in accordance with the mesh refinement, and also compared with the error of the solutions to the same problems obtained from the finite difference method (FDM). In this work, the FEM was more appropriate not only for its greater accuracy, but also for its versatility when dealing with multi-connected and irregular domains. Comparing variants of the FEM, again the GFEM is most useful when one wants only obtain the solution to $T(x,y,z)$, to be efficient and present a low computational cost, and LSFEM, is the most indicated for the calculation of the first derivative T. The accuracy in calculating the first derivative of T using LSFEM, in this study, gets to be three orders higher than the accuracy of results obtained from GFEM. The obtaining of the value of the first derivative of T is important in heat transfer studies, since it allows the analysis of the heat flow at any point of the domain.

The linearization technique used in this study is discussed by Campos et al. (2014) for solving a non-linear transient diffusive-convective problem in three dimensions using FDM with order high accuracy. In this work, the authors applied the Crank-Nicolson technique to discretize the time. The technique is derivatives from the weighted average over time of the independent variable in two consecutive time steps by linear interpolation of the variable values, where the coefficients of these derivatives value is 0.5. To linearize the convective terms, the Newton method is applied, since it is a simple technique that optimizes the calculation because it does not require an iterative linearization in each step of time, decreasing your computing time. The numerical solution obtained was using a regular mesh and compared with the exact solution of equation. The L_∞ errors were calculated for each refinement made in the mesh and a table was presented, containing the computational time and the convergence rate. Presented in tables, conform mesh is refined, the more accurate the result becomes, because the oscillations of numerical solution are softened.

The same discretization time and linearization techniques used in Campos et al. (2014) are used in the work done by Cruz et al. (2014). This work solves the Burgers equation in one dimension using finite difference method in four types of applications. The results were compared with the exact solutions and with the results obtained into other works. Again, the techniques present satisfactory results making the solution of the problem more accurate.

The main objective of this work is to apply the finite element method in LSFEM variant united with the Newton method for linearization of the nonlinear term of the 1D Burgers equation and solve it numerically.

METHODOLOGY

Model equation

Introduced here is a numerical study of the partial differential equation of the flow phenomenon in one direction without pressure gradient (equation Burgers 1D) defined in the domain $\Theta = \Xi \otimes \Omega$, $\Xi \subset \Re$, $\Omega \subset \Re$, in which Ξ and Ω are limited and closed domains, described as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \upsilon\frac{\partial^2 u}{\partial x^2} \tag{1}$$

In which υ is the cinematic viscosity (m/s^2) and $u = u(x,t)$ is the flow field in the x direction.

Weighted residual method

In this work, the aim is to use the weighted residual method to obtain an approximate solution to the differential Equation (1). For it will be used functions attempts type:

$$u \approx u_1(x,t), \tag{2}$$

So it can be defined an R residue as:

$$R \approx \frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} - \upsilon\frac{\partial^2 u}{\partial x^2} \tag{3}$$

Soon after, it introduces a set of weighting functions v_i (i = 1, 2,..., N_{nodes}) and defines an inner product (R,v_i). Then it is determined that the inner product is zero:

$$\int_\Omega R v_i \, d\Omega = 0 \tag{4}$$

Which is equivalent to force the error approximation of the differential equation is equal to zero on average. There are several ways to choose weighting functions v_i. This work will use the least squares method.

Least squares method

To obtain the approximate solution of the problem (1) we will use the least squares method, for which basic idea is to determine $u^e \in V^e$ for minimizing the integral of the square of the residue defined by Equation (3).
To this end, a quadratic functional is defined:

$$I(u_1) = \| R(u_1) \|^2 = \int_{\Omega^e} \left\{ \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} - \upsilon \frac{\partial^2 u_1}{\partial x^2} \right\}^2 d\Omega \quad (5)$$

For all $u_1 \in H^1$, in which H^1 is the Hilbert space of order 1.
The condition necessary so that $u_1 \in V$ be a minimizer of the functional I in Equation (5) is that in first variation of u_1 result:

$$\delta I(R) = 2\int_{\Omega} (\delta R) R d\Omega = 0 \text{ or } \int_{\Omega} (\delta R) R d\Omega = 0 \quad (6)$$

Note that when comparing the Equations (4) and (6) with that for the least squares method the weighting function is taken as the first variation of the residue.
The algebraic system generated by the formulation by the least squares method is symmetric and positive definite for any values of coefficients of Equation (1) in accordance with Jiang (1998). After formulation the finite element method, a linear system is generated, and from a computational code, the results for the velocity fields for each step time are found. Just for facility, we will use u instead of u_1.

FORMULATION BY LEAST SQUARES METHOD

Rearranging the Equation (1), separating the derivatives in time and space, the following is obtained:

$$\frac{\partial u^{n+1}}{\partial t} = \upsilon \frac{\partial^2 u^{n+1}}{\partial x^2} - u^{n+1} \frac{\partial u^{n+1}}{\partial x} \quad (7)$$

Here, we use the Newton method (Jiang, 1998) to linearize the non-linear term, as follows:

$$u^{n+1} \frac{\partial u^{n+1}}{\partial x} \cong u^{n+1} \frac{\partial u^n}{\partial x} + u^n \frac{\partial u^{n+1}}{\partial x} - u^n \frac{\partial u^n}{\partial x} \quad (8)$$

Substituting Equation (8) into Equation (7) is obtained:

$$\frac{\partial u^{n+1}}{\partial t} = \upsilon \frac{\partial^2 u^{n+1}}{\partial x^2} - u^n \frac{\partial u^{n+1}}{\partial x} - u^{n+1} \frac{\partial u^n}{\partial x} + u^n \frac{\partial u^n}{\partial x} \quad (9)$$

Then, Equation (9) discretized the transient term, which in this work used Crank-Nicolson method resulting in the following equation:

$$\frac{u^{n+1} - u^n}{\Delta t} = 0.5\left(\upsilon \frac{\partial^2 u^{n+1}}{\partial x^2} - u^n \frac{\partial u^{n+1}}{\partial x} - u^{n+1} \frac{\partial u^n}{\partial x} + u^n \frac{\partial u^n}{\partial x} \right) + 0.5\left(\upsilon \frac{\partial^2 u^n}{\partial x^2} - u^n \frac{\partial u^n}{\partial x} - u^n \frac{\partial u^n}{\partial x} + u^n \frac{\partial u^n}{\partial x} \right) \quad (10)$$

Simplifying and rearranging the equation (10), leads to:

$$0.5\upsilon \frac{\partial^2 u^{n+1}}{\partial x^2} + A \frac{\partial u^{n+1}}{\partial x} + B u^{n+1} = -C \quad (11)$$

Where, $A = -0.5 u^n$; $B = -\frac{1}{\Delta t} - 0.5 \frac{\partial u^n}{\partial x}$; $C = \frac{u^n}{\Delta t} + 0.5\upsilon \frac{\partial^2 u^n}{\partial x^2}$

From Equation (11) the following residual equation can be written:

$$R \cong 0.5\upsilon \frac{\partial^2 u^{n+1}}{\partial x^2} + A \frac{\partial u^{n+1}}{\partial x} + B u^{n+1} + C \quad (12)$$

and taking an approximation in the form:

$$u = \sum_{i=1}^{Nnodes} u_i N_i \quad (13)$$

Where N_i is the interpolation functions, obtaining the expression:

$$R \cong 0.5\upsilon \sum_{i=1}^{Nnodes} \frac{\partial^2 N_i}{\partial x^2} u_i^{n+1} + A \sum_{i=1}^{Nnodes} \frac{\partial N_i}{\partial x} u_i^{n+1} + B \sum_{i=1}^{Nnodes} N_i u_i^{n+1} + C \quad (14)$$

Before using the results presented in Equation (6) it is necessary to define the first variation of the residue (Equation 14) is as follows:

$$\delta R = \frac{\partial R}{\partial u^{n+1}} \delta u^{n+1} \quad (15)$$

Where

$$\frac{\partial R}{\partial u^{n+1}} = 0.5\upsilon \frac{\partial^2 N_i}{\partial x^2} + A \frac{\partial N_i}{\partial x} + B N_i \quad (16)$$

And thus, substituting the Equations (14), (15) and (16) into Equation (6), the following is obtained:

$$[\delta u^{n+1}] \int_{\Omega} \left[0.5\upsilon \frac{\partial^2 N_i}{\partial x^2} + A \frac{\partial N_i}{\partial x} + B N_i \right] \left[0.5\upsilon \sum_{j=1}^{Nnodes} \frac{\partial^2 N_j}{\partial x^2} u_j^{n+1} + A \sum_{j=1}^{Nnodes} \frac{\partial N_j}{\partial x} u_j^{n+1} + B \sum_{j=1}^{Nnodes} N_j u_j^{n+1} + C \right] d\Omega = 0 \quad (17)$$

Starting from the hypothesis that $\delta u^{n+1} \neq 0$, it will be possible generate the following matrix vetor system:

$$[A]\{u^{n+1}\} = \{B\} \quad (18)$$

Where

$$A_{ij} = \int_{\Omega} \left[0.5\upsilon \frac{\partial^2 N_i}{\partial x^2} + A \frac{\partial N_i}{\partial x} + B N_i \right] \left[0.5\upsilon \frac{\partial^2 N_j}{\partial x^2} + A \frac{\partial N_j}{\partial x} + B N_j \right] d\Omega = 0 \quad (19)$$

$$B_i = -\int_\Omega \left[0.5\upsilon \frac{\partial^2 N_i}{\partial x^2} + A \frac{\partial N_i}{\partial x} + BN_i \right] \cdot Cd\Omega = 0 \tag{20}$$

Considering our mesh composed of three elements called nodes *m*, *n* and *p* (with equal spacing between these nodes) in each element, and the following weighting functions:

$$N_m = \frac{2}{l^2}[x^2 - (x_n + x_p)x + x_n x_p]$$

$$N_n = -\frac{4}{l^2}[x^2 - (x_m + x_p)x + x_m x_p]$$

$$N_p = \frac{2}{l^2}[x^2 - (x_m + x_n)x + x_m x_n]$$

will be constructed a matrix vetor system of each element as follows:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} \tag{21}$$

Emphasizing that $A_{21} = A_{12}$, $A_{31} = A_{13}$ and $A_{32} = A_{23}$ where

$$A_{11} = \frac{4}{l^4}\left[\frac{B_m^2}{5}(x_p^5 - x_m^5) + \frac{B_m\alpha}{2}(x_p^4 - x_m^4) + \frac{(2B_m\beta + \alpha^2)}{3}(x_p^3 - x_m^3) + \alpha\beta(x_p^2 - x_m^2) + \beta^2(x_p - x_m) \right]$$

With $\alpha = 2A_m - B_m(x_n + x_p)$ and $\beta = \upsilon - A_m(x_n + x_p) + B_m x_n x_p$.

$$A_{12} = \frac{8}{l^4}\left[-\frac{B_m B_n}{5}(x_p^5 - x_m^5) + \frac{(B_m\gamma - B_n\alpha)}{4}(x_p^4 - x_m^4) + \frac{(B_m\varepsilon + \alpha\gamma - B_n\beta)}{3}(x_p^3 - x_m^3) + \frac{(\alpha\varepsilon + \beta\gamma)}{2}(x_p^2 - x_m^2) + \beta\varepsilon(x_p - x_m) \right]$$

With $\gamma = -2A_n + B_n(x_m + x_p)$ and $\varepsilon = -\upsilon + A_n(x_m + x_p) - B_n x_m x_p$.

$$A_{13} = \frac{4}{l^4}\left[\frac{B_m B_p}{5}(x_p^5 - x_m^5) + \frac{(B_m\theta + B_p\alpha)}{4}(x_p^4 - x_m^4) + \frac{(B_m\lambda + \alpha\theta + B_p\beta)}{3}(x_p^3 - x_m^3) + \frac{(\alpha\lambda + \beta\theta)}{2}(x_p^2 - x_m^2) + \beta\lambda(x_p - x_m) \right]$$

with $\theta = 2A_p - B_p(x_m + x_n)$ and $\lambda = \upsilon - A_p(x_m + x_n) + B_p x_m x_n$.

$$A_{22} = \frac{16}{l^4}\left[\frac{B_n^2}{5}(x_p^5 - x_m^5) - \frac{B_n\gamma}{2}(x_p^4 - x_m^4) + \frac{(-2B_n\varepsilon + \gamma^2)}{3}(x_p^3 - x_m^3) + \gamma\varepsilon(x_p^2 - x_m^2) + \varepsilon^2(x_p - x_m) \right]$$

$$A_{23} = \frac{8}{l^4}\left[-\frac{B_n B_p}{5}(x_p^5 - x_m^5) + \frac{(-B_n\theta + B_p\gamma)}{4}(x_p^4 - x_m^4) + \frac{(-B_n\lambda + \gamma\theta + B_p\varepsilon)}{3}(x_p^3 - x_m^3) + \frac{(\gamma\lambda + \varepsilon\theta)}{2}(x_p^2 - x_m^2) + \varepsilon\lambda(x_p - x_m) \right]$$

$$A_{33} = \frac{4}{l^4}\left[\frac{B_p^2}{5}(x_p^5 - x_m^5) + \frac{B_p\theta}{2}(x_p^4 - x_m^4) + \frac{(2B_p\lambda + \theta^2)}{3}(x_p^3 - x_m^3) + \theta\lambda(x_p^2 - x_m^2) + \lambda^2(x_p - x_m) \right]$$

$$B_1 = -\frac{2C_m}{l^2}\left[\frac{B_m}{3}(x_p^3 - x_m^3) + \frac{\alpha}{2}(x_p^2 - x_m^2) + \beta(x_p - x_m) \right]$$

$$B_2 = -\frac{4C_n}{l^2}\left[-\frac{B_n}{3}(x_p^3 - x_m^3) + \frac{\gamma}{2}(x_p^2 - x_m^2) + \varepsilon(x_p - x_m) \right]$$

$$B_3 = -\frac{2C_p}{l^2}\left[\frac{B_p}{3}(x_p^3 - x_m^3) + \frac{\theta}{2}(x_p^2 - x_m^2) + \lambda(x_p - x_m) \right]$$

NUMERICAL APPLICATIONS

From the matrix vetor system presented in Equation (21) is constructed for all the elements of the computational mesh an overall linear system from a computational code written in Fortran language. To solve the global linear system in each step of time is used the Gauss-Seidel method with stop error equal to 10^{-6}. To analyze the efficiency of the proposed formulation presents two applications with exact solution for comparison.

Application 1

In this application, considering Re = 1, was used the exact solution $u(x,t) = 2x/(1+2t)$ (Cruz et al., 2014) for comparison with the numerical results. After several refinements in space (Δx) and in time (Δt), for this application the proposed formulation present a clear convergence range. As can be seen in Tables 1 and 2, the numerical solution showed an efficiency of at less 10^{-2} in mesh where the relation $\Delta t/(\Delta x)^2$ is between 0.25 and 0.45 (aproximately). It is important to note that several others combinations between Δx and Δt were tested validating the found interval, however these results were not presented because of table size.

Application 2

In this application will be used as boundary conditions $u(0,t) = u(1,t) = 0$ and as initial condition $u(x,0) = 4x(1 - x)$ (Cruz et al., 2014; Hassanien et al., 2005; Ozis et al., 2003). In the Application 2, as can be seen from Tables 3 and 4, again there arises a convergence range of the proposed formulation, when analyzed numerical results consistent with that presented in Cruz et al. (2014), Hassanien et al. (2005) and Ozis et al. (2003) for $x = 0.25$ and $t = 0.1$ ($u(0.25, 0.1) = 0.26245$, chosen point for this study).

Conclusions

The LSFEM proved extremely effective in solving the

Table 1. Several results of the L_∞ norm for different refinements.

Time steps	Number of elements						
	9	10	11	12	13	14	15
720	4.12E+02						
760	6.25E+00						
800	6.51E-04	9.55E+02					
920	7.57E-04	7.48E+02					
960	8.04E-04	5.17E-04					
1000	8.73E-04	5.34E-04					
1040	9.53E-04	5.54E-04					
1080	1.05E-03	5.76E-04	4.62E+02				
1140	1.28E-03	6.18E-04	8.26E+00				
1160	1.38E-03	6.34E-04	4.28E-04				
1320	5.07E-03	8.32E-04	4.83E-04	5.63E+02			
1360	1.83E-02	9.13E-04	5.01E-04	8.51E+00			
1400	1.00E+03	1.01E-03	5.23E-04	3.63E-04			
1440	1.35E+03	1.15E-03	5.47E-04	3.72E-04			
1560		2.04E-03	6.46E-04	4.03E-04	1.59E+03		
1600		2.83E-03	6.92E-04	4.16E-04	8.63E+00		
1640		4.74E-03	7.46E-04	4.30E-04	3.09E-04		
1680		1.50E-02	8.13E-04	4.45E-04	3.15E-04		
1720		1.10E+03	8.59E-04	4.63E-04	3.22E-04		
1760		1.24E+03	1.00E-03	4.83E-04	3.29E-04		
1800			1.14E-03	5.06E-04	3.37E-04	6.83E+02	
1840			1.33E-03	5.32E-04	3.45E-04	5.55E+02	
1880			1.61E-03	5.62E-04	3.55E-04	2.64E-04	
2040			1.68E-02	7.41E-04	4.02E-04	2.83E-04	1.11E+03
2080			2.19E+03	8.12E-04	4.18E-04	2.89E-04	1.74E+03
2120			6.87E+02	8.99E-04	4.35E-04	2.96E-04	1.24E+01
2160				1.01E-03	4.54E-04	3.02E-04	2.30E-04
2400				5.75E-03	6.39E-04	3.57E-04	2.54E-04
2440				2.79E-02	6.90E-04	3.69E-04	2.58E-04
2480				1.69E+03	7.52E-04	3.82E-04	2.63E-04
2520				6.94E+02	8.29E-04	3.97E-04	2.68E-04
2560					9.24E-04	4.13E-04	2.74E-04
2800					3.72E-03	5.66E-04	3.21E-04
2880					1.11E-01	6.53E-04	3.42E-04
2920					1.54E+03	7.10E-04	3.54E-04
2960					2.03E+03	7.78E-04	3.67E-04
3080						1.11E-03	4.16E-04
3320						1.88E-02	5.82E-04
3360						2.11E+03	6.27E-04
3400						1.87E+03	6.81E-04
3480							8.24E-04
3760							4.03E-03
3800							9.84E-03
3880							1.60E+03

problem nonlinear transient proposed, but it is important to note the convergence range to which the LSFEM in junction with the Newton method (linearization) presents. What should be highlighted is that the Newton method proposed here is only an iterative step in each time step, and this has led to this convergence range, but a

Table 2. Different values for relation $\Delta t/(\Delta x)^2$ for different refinements.

Time steps	Number of elements						
	9	**10**	**11**	**12**	**13**	**14**	**15**
720	0.501						
760	0.475						
800	0.451	0.551					
920	0.392	0.479					
960	0.376	0.459					
1000	0.361	0.441					
1040	0.347	0.424					
1080	0.334	0.408	0.490				
1140	0.317	0.387	0.464				
1160	0.311	0.380	0.456				
1320	0.273	0.334	0.401	0.473			
1360	0.265	0.324	0.389	0.460			
1400	0.258	0.315	0.378	0.446			
1440	0.251	0.306	0.367	0.434			
1560		0.283	0.339	0.401	0.467		
1600		0.276	0.331	0.391	0.456		
1640		0.269	0.323	0.381	0.445		
1680		0.263	0.315	0.372	0.434		
1720		0.256	0.308	0.363	0.424		
1760		0.251	0.301	0.355	0.414		
1800			0.294	0.347	0.405	0.467	
1840			0.288	0.340	0.396	0.457	
1880			0.281	0.332	0.388	0.447	
2040			0.259	0.306	0.357	0.412	
2080			0.254	0.300	0.350	0.404	0.462
2120			0.250	0.295	0.344	0.397	0.453
2160				0.289	0.338	0.389	0.445
2400				0.260	0.304	0.350	0.400
2440				0.256	0.299	0.345	0.394
2480				0.252	0.294	0.339	0.388
2520				0.248	0.289	0.334	0.381
2560					0.285	0.329	0.375
2800					0.260	0.300	0.343
2880					0.253	0.292	0.334
2920					0.250	0.288	0.329
2960					0.246	0.284	0.325
3080						0.273	0.312
3320						0.253	0.289
3360						0.250	0.286
3400						0.247	0.283
3480							0.276
3760							0.256
3800							0.253
3880							0.248

proposal for future work is to use the LSFEM with the Newton method with an iterative process at each time step and analyze how it behaves this convergence range.

In this work since it is a one-dimensional problem in its construction of the computational code a very important feature of problems solved with LSFEM was not

Table 3. Several results of the L_∞ norma for different refinements.

Time steps	Number of elements								
	32	**34**	**36**	**38**	**40**	**42**	**44**	**46**	**48**
900	2.53E+01								
1000	1.08E-03								
1100	1.10E-03	1.07E+01							
1200	1.13E-03	4.58E-05	1.75E+01						
1300	1.17E-03	2.38E-04	1.06E-03	1.26E+03					
1400	1.24E-03	4.96E-04	1.07E-03	2.53E-03					
1500	1.37E-03	8.63E-04	1.09E-03	1.55E-04	8.65E+02				
1600	1.69E-03	1.42E-03	1.12E-03	3.52E-05	1.04E-03	1.22E+02			
1700	3.70E-03	2.40E-03	1.15E-03	1.15E-04	1.05E-03	3.89E-04			
1800	9.62E+02	4.52E-03	1.20E-03	3.12E-04	1.06E-03	3.25E-04	8.69E+02		
1900		1.25E-02	1.28E-03	5.77E-04	1.07E-03	2.51E-04	1.03E-03		
2000		1.99E+03	1.45E-03	9.57E-04	1.09E-03	1.62E-04	1.03E-03	6.52E+00	
2100			1.97E-03	1.54E-03	1.11E-03	5.34E-05	1.04E-03	4.59E-04	
2200			2.51E-02	2.58E-03	1.14E-03	8.17E-05	1.05E-03	4.12E-04	3.11E+01
2300			4.07E+03	4.89E-03	1.19E-03	2.54E-04	1,06E-03	3.28E-04	1.02E-03
2400				1.46E-02	1.27E-03	4.84E-04	1,07E-03	2.95E-04	1.02E-03
2500				4.25E+03	1.41E-03	8.04E-04	1.08E-03	2.21E-04	1.03E-03
2600					1.81E-03	1.28E-03	1.10E-03	1.32E-04	1.03E-03
2700					6.19E-03	2.06E-03	1.12E-03	2.42E-05	1.04E-03
2800					3.56E+03	3.60E-03	1.16E-03	1.10E-04	1.05E-03
2900						8.01E-03	1.21E-03	2.83E-04	1.05E-03
3000						1.37E-01	1.30E-03	5.13E-04	1.06E-03
3100						2.90E+03	1.48E-03	8.33E-04	1.08E-03
3200							2.08E-03	1.31E-03	1.10E-03
3300							4.53E-01	2.09E-03	1.12E-03
3400							3.70E+03	3.63E-03	1.15E-03
3500								8.05E-03	1.20E-03
3600								1.39E-01	1.27E-03
3700								2.97E+03	1.42E-03
3800									1.82E-03
3900									6.26E-03
4000									4.92E+03

Table 4. Different values for relation $\Delta t/(\Delta x)^2$ for different refinements.

Time steps	Number of elements								
	32	**34**	**36**	**38**	**40**	**42**	**44**	**46**	**48**
900	0.469								
1000	0.423								
1100	0.384	0.433							
1200	0.352	0.397	0.444						
1300	0.325	0.366	0.410	0.456					
1400	0.302	0.340	0.381	0.424					
1500	0.282	0.317	0.355	0.395	0.437				
1600	0.264	0.298	0.333	0.371	0.410	0.452			
1700	0.249	0.280	0.313	0.349	0.386	0.425			
1800	0.235	0.265	0.296	0.329	0.365	0.401	0.440		
1900		0.251	0.280	0.312	0.345	0.380	0.417		

Table 4. Contd.

2000	0.238	0.266	0.296	0.328	0.361	0.396	0.432	
2100		0.254	0.282	0.312	0.344	0.377	0.412	
2200		0.242	0.270	0.298	0.328	0.360	0.393	0.428
2300		0.232	0.258	0.285	0.314	0.344	0.376	0.409
2400			0.247	0.273	0.301	0.330	0.360	0.392
2500			0.237	0.262	0.289	0.317	0.346	0.376
2600			0.228	0.252	0.278	0.305	0.333	0.362
2700				0.243	0.268	0.293	0.320	0.348
2800				0.234	0.258	0.283	0.309	0.336
2900					0.249	0.273	0.298	0.324
3000					0.241	0.264	0.288	0.314
3100					0.233	0.256	0.279	0.304
3200						0.248	0.270	0.294
3300						0.240	0.262	0.285
3400						0.233	0.254	0.277
3500							0.247	0.269
3600							0.240	0.261
3700							0.234	0.254
3800								0.248
3900								0.241
4000								0.235

highlighted: Its formulation creates a positive-definite and symmetric matrix, which enables the use of a method of conjugated gradients which has a more significant convergence than the Gauss-Seidel method and the possibility of economy in the storage of non-zero coefficients to have a symmetric matrix. The authors believe these characteristics indicate that the LSFEM + Newton method is a good option for solution of nonlinear transient problems and in areas two and three dimensional.

Conflict of Interest

The authors have not declared any conflict of interest.

ACKNOWLEDGEMENTS

The National Council of Scientific Development and Technology, CNPq, Brazil (Proc. 408250/2013-5) and FAPESP (Proc. 2014/06679-8) supported the present work.

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